DIVERSIFICATION, CAPITAL GAINS TAXES AND LONG TERM RETURN

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Suppose an investor owns a portfolio consisting of a single (small group of) stock(s) that has performed well. Should the investor enter a diversification program; i.e., sell some or all of the assets in the original portfolio, in order to increase diversification by investing in a more highly diversified portfolio?

In spite of the significant losses in portfolio value due to the payment of capital gains taxes and the likelihood that a new, more market-like, portfolio may not have as high an expected return, many financial consultants and personal trust officers often recommend such a plan. But how can such an investment program be justified?

The benefits of diversification are generally associated with reducing a portfolio's exposure to losses.[1] By reducing risk unrelated to the market, portfolio returns can be confined to a narrower range and the probability of low return in any given period is reduced. However, diversification by itself does not increase expected return. On a short term basis an investor may pay a substantial price for an increase in portfolio diversification.

The paradox of theory with practice can be removed by changing the focus from single-period or short term return to multiperiod or long term return. It will be shown that a diversification program over time may lead not only to substantially lower portfolio volatility and lower probability of exposure to loss but also a higher expected growth rate than the original portfolio even when capital gains taxes are taken into account.

PROBLEM STATEMENT

A diversification program is defined in terms of: a) the proportion, D, of initial portfolio wealth that will be sold and reinvested (diversified) over the N-period investment horizon; b) the number, K, $K \leq N$, of successive periods over which equal amounts of D will be diversified; c) the proportion P, the marginal capital appreciation tax rate. All other transaction costs are ignored or are subsumed in the tax factor. In general, we will assume that the original portfolio (O) is significantly less well diversified than the target portfolio (T), $r_O < r_T$, and has higher expected return in the form of higher systematic risk or beta, $B_O > B_T$.

It is assumed for convenience that financial markets are (strong form) efficient[2] so that the basic determinants of portfolio return are its risk characteristics (systematic and diversifiable risk levels) and tax effects. Note that an efficient market framework raises, rather than limits, the practical value of the results. The description of a diversification program that can be justified in a strong form efficient market context does not limit any of the benefits that may be added via active management of the assets of the diversified portfolio.

The long term consequences of diversification leads naturally to consideration of properties of geometric mean (compound, growth rate) return which is the appropriate measure of portfolio return over time. The N-period geometric mean return is defined as:

$$G_N=((1+R_1)\ (1+R_2)\ \ldots\ (1+R_N))^{1/N} - 1 \qquad (1)$$

where $R_i > -1$, $i=1, \ldots, N$, is the single-period return in the ith period of the N-period investment horizon. For purposes of convenience we assume that cash flows are absent and returns reinvested and that the return distribution is intertemporally independent and identically distributed over time.

Numerous authors have examined the investment properties of the geometric mean return distribution as a portfolio selection criterion.[3] In particular, the maximization of the expected geometric mean leads, over a sufficiently long investment horizon, almost surely to more wealth than any other essentially different investment strategy (Thorp, 1974).

Some important objections have been raised to the geometric mean portfolio objective. In particular, Samuelson and Merton (1974) have shown that maximization of the expected geometric mean is not consistent with expected utility maximization under a wide variety of conditions. It is beyond the scope of this paper to investigate the foundations of investment criteria for institutional portfolio management.[4] From the perspective of practical portfolio management, the expected value of the portfolio growth rate over an investment horizon is often an important part of stated investment objectives and ex post investment performance evaluation. As importantly, we avoid the unnecessary limitation of considering only the portfolio that maximizes the expected geometric mean over an infinite horizon.[5] The parameters of the geometric mean distribution are used as a guide for choosing those portfolios that have attractive near and long term investment characteristics. Used with an awareness of its limitations, the criterion may be useful for investors with investment objectives consistent with properties of the criterion.

SOME THEORETICAL RESULTS ON LONG TERM RETURN.

Before we illustrate the diversification issues further, an important, but not particularly intuitive, property of the expected geometric mean must be described. Michaud (1981) showed mathematically that the expected geometric mean will generally decline in value as the number of periods, or length of the horizon, increases.[6] Some intuition for this result may be found in the following observation: The expected geometric mean (generally) acts as if the active placement of assets at risk in each additional time period implies the assumption of additional risk or probability of loss with respect to the option of not placing the assets at risk.

It is well known that the expected geometric mean return is a negative function (beyond the first period) of the portfolio variance with a well defined limit of $(e^{E(\ln(1+r)} - 1)$. Since portfolio diversification reduces the total risk of the portfolio, diversification increases the expected geometric mean. Therefore, if two portfolios have equal (single-period) expected returns, the expected growth rate of the more well diversified portfolio will be greater. The interesting practical issue is whether, in the long run, a better diversified portfolio with less (single-period) expected return, has a larger expected growth rate.

Some of these relationships are illustrated in Figure 1, where portfolio T has the same (single period) expected return as portfolio O but higher diversification. Consequently, portfolios T and O have the same intercept but the expected growth rate for portfolio O decreases more significantly as a function of the length of the investment horizon. It should be noted that, as previously described, the long term expected geometric mean of each portfolio has a well defined limit that is not necessarily zero or negative.

SOME SIMPLE SPECIAL CASES

A simple but important special case of the diversification problem consists of the sale of all the assets in the original portfolio at the beginning of the first period and the after-tax remainder, 1-P invested in the target portfolio (D=1, K=1). The wealth ratio at time N is:

$$(2) \qquad W_N = W_o\ (1 - P) \prod^{N} (1 + R_i).$$

By definition, the geometric mean return in the Nth period, $G_N(R)$, where R represents the vector of N-period returns, is:

$$(3) \qquad G_N(R) = (1-P)^{1/N} \{ \prod^{N} (1 + R_i) \}^{1/N} - 1.$$

The term $(1-P)^{1/N}$ in (3) describes the effect of loss of wealth on the expected growth rate over time. As N increases this factor approaches one. Consequently, the loss of wealth which occurs in the first period, due to capital gains taxes, has a diminishing effect on the expected growth rate of the portfolio as N increases. As the loss of wealth factor diminishes, the expected long term rate of return will increasingly depend on the risk-return characteristics of the target portfolio and approach the long term limit of the expected growth rate of the target portfolio.

Figure 2 illustrates the results of this argument.[7] As in Figure 1, we compare the expected geometric mean over time for two portfolios O -- the original, less well diversified portfolio and T -- the target or better diversified portfolio. We will also assume that P = 0.35 or that portfolio T is 65% of the value of portfolio O at the beginning of the first period due to capital gains taxes.

The effect of taxes on return, using the value of the portfolio O as the base for the computation, are most severe in the first period. As observed from the increase in the expected geometric mean for portfolio T, the effects of the initial loss of wealth diminish over time. The higher long term rate of return for portfolio T is indicative of the higher level of portfolio diversification.

After a number of periods, the expected geometric mean return of both portfolios may coincide. This point is called the "crossover." From this point on, the diversified "target" portfolio has higher expected compound return with greatly reduced risk.

The crossover point is an important focus of a diversification program analysis. The investment environment assumptions, which will be detailed later, can have an important effect on the results depicted in Figure 2, especially on the value and even the existence of a crossover point.

Using approximation methods[8] an analysis of the diversification problem for the case when all assets, D, that are to be diversified are sold in the first period (K=1) can be derived. The solution is essentially a properly weighted average of the expected geometric means of the original and target portfolios. Figure 3, illustrating the approximation analysis, shows that a smaller proportion of the sale of the original asset results in less of a loss in portfolio return due to a loss of portfolio value and the likelihood of an earlier crossover point for the strategy but a lower long term growth rate than the target portfolio.

MONTE CARLO SIMULATION ANALYSIS

Although Figures 1, 2 and 3 bracket many of the major characteristics of the diversification problem, the question of diversifying a portion of the original assets over many periods has not been dealt with. This factor does not readily lend itself to analytic description.

Monte Carlo simulation was used to study this important aspect of the diversification problem. This is a statistical technique often used for understanding the implications of any given set of portfolio and market assumptions on portfolio performance.[9] Simulating the behavior of the portfolio over time can determine the likely return, ranges of normal variation, and the distribution of returns, under a given set of assumptions. By varying key assumptions we may observe the effect of the market environment and diversification strategies to determine appropriate and optimal strategies with respect to investor objectives and constraints.

In the appendix, a detailed description of the simulation procedure that is used as the basis of the simulation studies is given. The market line model is used to generate portfolio return based on the risk characteristics of the original and target portfolios with respect to the market assumptions and the diversification program. The capital market parameters that describe the capital market environment consist of an expected market risk premium and market volatility (standard deviation).

The two market environment assumptions used in the simulations are:

Mkt	Risk premium	St. dev.
Average	7%	20%
Volatile	7%	30%

The "average" market parameters represent a reasonably typical set of assumptions based on historical data; the "volatile" market parameters represent a more pessimistic and risky market environment.

The portfolio risk-diversification parameters for the original and target portfolios are:

Portfolio	Beta	Correlation with market
O	B = 1.2	r = 0.60.
T	B = 1.0	r = 0.95.

In many cases of interest, the original portfolio will have significantly lower portfolio diversification, and higher beta, than our assumptions. This is in line with the generally conservative tilt of our assumptions.

Since the market assumptions are given in annual terms, the compounding periods of the simulation studies should be interpreted as years. Two hundred simulations were used for all simulation studies.

Figure 4 describes the results of the Monte Carlo simulation. The median portfolio growth rate for the indicated five proportions of original asset diversification are plotted over twenty years. Apart from statistical variation that is a natural part of the simulation process, the results of Figures 3 and 4 are, as expected, similar. Figure 4 serves as a benchmark for the portfolio simulation analysis of other aspects of the diversification process.

The effect of asset diversification in a more highly volatile market is shown in Figure 5. The assumptions of Figure 5 are exactly those of Figure 4 except for an increase in market volatility. The long term median compound rate of return for the 0% asset diversified portfolio is quite low. The beneficial effects of asset diversification are dramatically apparent in a more volatile market, with a much shorter crossover time.

In Figure 6, we examine the effect of asset diversification over a number of periods, for the assumptions in Figure 4. In contrast to the situation of Figure 4 where all asset diversification takes place at the beginning of the first period, the indicated percent of asset diversification takes place, in equal amounts, over ten periods. The net effect is to reduce the short term losses due to capital gains taxes to a much more acceptable level. Asset diversification over a number of periods provides the investor with minimum short term losses with respect to the returns that would be anticipated from the original holdings while building up portfolio diversification and its beneficial long term effects. The long run median rate of return closely resembles the anticipated return from the 100% asset diversified portfolio.

In Figure 7, we examine the same diversification process as in Figure 6, for the more volatile market case. The market volatility affects judgment of an appropriate rate of asset diversification. In highly volatile markets, quick diversification of assets is desirable, while a more leisurely pace is appropriate in less volatile markets.

In Figure 8, an alternative way of presenting the effects of asset diversification on portfolio performance is shown which focuses on portfolio volatility. The assumptions are those of Figure 4. The distribution of compound return is presented for the fifth year and is graphed versus the level of asset diversification. The horizontal axis consists of five levels of diversification, from 0% to 100% in steps of 25%.

The middle curve represents median compound return for each level of asset diversification in the fifth year. The median curve points out that, as the level of asset diversification is increased, the anticipated level of compound return in the fifth year decreases. The top curve is the 95% percentile of compound returns and represents the point such that, only five percent of returns are higher. The 75th, 25th, and 5th percentile curves are defined similarly. The 5th and 95th percentile curves define a reasonable range of compound portfolio return. Figure 8 shows that the variability or range of returns and consequently the probability of low returns is greatly reduced by asset diversification.

In Figure 9, the effect of asset diversification is shown, with respect to the assumptions in Figures 4 and 8, for the twentieth year. On a long term basis, diversification tends to increase expected compound return while greatly reducing exposure to the possibility of low returns. It is of interest to note that, for the assumptions of Figure 9, the probability of negative returns is reduced by asset diversification to virtually zero.

SUMMARY

The major conclusions of the diversification study are:

1. Portfolio diversification will often increase the expected portfolio growth rate and significantly reduce the portfolio's exposure to loss even after consideration of the loss of wealth due to capital gains taxes and the lower expected return of the more highly diversified portfolio.

2. In many cases, the inferiority of the performance of diversified portfolios may be relatively short lived. The risk-return characteristics of the target portfolio often dominate portfolio return within relatively short time horizons.

3. The more volatile the market environment, the more attractive is the benefit of a diversification program over time.

4. Under "average market" conditions, the diversification program with the most attractive characteristics appears to be one that diversifies a significant proportion of assets over a number of periods. This is because short run portfolio returns closely resemble anticipated returns from the original portfolio while long run return closely resembles the long term expected return of the highly diversified portfolio.

Although no attempt was made to determine diversification objectives with respect to anticipated revenue flows, this would generally be an important part of a diversification study tailored to meet the needs of particular investors or institutions. It should also be noted that the policy of holding the level of portfolio risk and diversification constant over time is not multiperiod optimal (see e.g., Michaud and Monahan, 1986).

In spite of the generally attractive portfolio return characteristics of diversification, it cannot be recommended for everyone. The basic tradeoff remains short term losses for long term gains. Alternatively, the tradeoff amounts to a reduced probability of loss which is offset by lower short term returns.

Also, the attractiveness of a diversification program on a long term basis is strongly affected by the assumptions which are made concerning the investment environment over the investment horizon. Under optimistic market assumptions, the crossover point, which may make long term diversification particularly attractive, may not exist. Therefore, investors with optimistic assumptions on the market may be badly advised to accept a reduction of portfolio return for the sake of diversification.

Ultimately, the appropriateness of any diversification program depends crucially on investment objectives and the need to make clear the consequences of a given set of assumptions and investor attitudes towards risk. However, the study clearly shows that, for investors with reasonably long term investment objectives, under reasonable market assumptions, diversification is an attractive characteristic of prudent investment management. In many cases, high capital gains taxes do not mitigate the attractiveness of diversification except to defer its benefits over a relatively short investment horizon.

APPENDIX

This appendix describes some of the technical details involved with a Monte Carlo simulation of the type used in the diversification study of this paper.

The "market line model" is:

$$R_i = R_f + B(R_{Mi} - R_f) + e_i \qquad (6)$$

where R_i is the total return of the portfolio in each period $i = 1,\ldots,N$, $R_i > -1$, R_f is the risk free rate in each period of the N period investment horizon, R_{Mi} is the return of the "Market" or benchmark portfolio in period i, the quantity $R_M - R_f$ is the market risk premium, B is the portfolio's "beta" or estimated level of systematic risk with respect to the benchmark portfolio, e_i is the return specific to the portfolio and unrelated to the market return in period i.

The portfolio's risk-diversification parameters are B and r; r is the correlation of the portfolio with the market portfolio.

The capital market parameters that are used to describe the investment environment over the N-period investment horizon are the expected market risk premium in each period, $E(R_M) - R_f$), the standard deviation of market returns in each period, sigM, and the risk free rate R_f. The assignment of the risk free rate value does not affect the simulation analysis.

If values for the portfolio risk-diversification parameters, and for the capital market parameters, are given, the risk-return parameters of the portfolio in each period can be determined by:

$$E(R) = R_f + B(E(R_M) - R_f) \qquad (7)$$

$$\mathrm{sig}(R) = B\,\mathrm{sig}_M/r$$

Returns are generated in each period for the original and target portfolio according to their risk-return characteristics as defined by (7). Returns are assumed to have a (left truncated) normal distribution; the IBM SSP Gauss subroutine was used to derive the random variates.

The three key parameters that describe a diversification program are D, K and P. Reference to R_i^O and R_i^T will denote respectively the total return in the ith period from the original and target portfolio.

D_i will be used to refer to the proportion of assets which will be diversified in the ith period of assets that remain at the beginning of the period. The formula for D_i given D and K is:

(8) $D_i = D/(K-(i-1)), \quad i = 1, \ldots, K$

The geometric mean return in period i has two forms depending on whether $i \leq K$ (diversification of assets is not complete) or $i > K$, (diversification of assets has been completed). The simulation program computes the following formula:

Given $i \leq K$, $G(_i(R) =$

$$\Big\{ \prod_{J=1}^{i}\{(1-D_J)(1+R_J^O)\} + \prod_{J=1}^{i-1}\{(1-D_J)(1+R_J^O)\}D_i(1-P)(1+R_i^T) +$$

$$\prod_{J=1}^{i-2}\{(1-D_J)(1+R_J^O)\}D_{i-1}(1-P)\prod_{J=i-1}^{i}(1+R_J^T) + \ldots +$$

$$(1-D_1)(1+R_1^O)D_2(1-P)\prod_{J=2}^{i}(1+R_J^T) + D(1-P)\prod_{J=1}^{i}(1+R_J^T) \Big\}^{1/i} - 1$$

(9)

Given $i > K$, $G_i(R) =$

$$\Big\{ \prod_{J=1}^{K}\{(1-D_J)(1+R_J^O)\}\prod_{J=K+1}^{i}(1+R_J^O) +$$

$$\prod_{J=1}^{K-1}\{(1-D_J)(1+R_J^O)\}D_K(1-P)\prod_{J=K}^{i}(1+R_J^T) +$$

$$\prod_{J=1}^{K-2}\{(1-D_J)(1+R_J^O)\}D_{K-1}(1-P)\prod_{J=K-1}^{i}(1+R_J^T) + \ldots +$$

$$(1-D_1)(1+R_1^O)D_2(1-P)\prod_{J=2}^{i}(1+R_J^T) + D_1(1-P)\prod_{J=1}^{i}(1+R_J^T) \Big\}^{1/i} - 1$$

The long term asset diversification problem amounts to the computation and analysis of equation (9).

FOOTNOTES

1 See e.g., MacDonald (1975).

2 See e.g., Fama (1970), Jensen, (1972).

3 Breiman, 1960; Hakansson, 1971a,b; Hakansson and Miller, 1975; Kelly, 1956; Latane, 1959; Markowitz, 1959, Ch. 6; Thorp, 1974.

4 Hakansson, 1979; Levy and Markowitz, 1979; Merton and Samuelson, 1974; Samuelson and Merton, 1974, Michaud 1981.

5 Markowitz, 1959, Ch. 6, noted that there may exist portfolios on the (single period) mean-variance efficient frontier with a smaller expected growth rate and more volatility than other portfolios on the efficient frontier. When the horizon is not infinite and the mean and variance of the single period return distribution is of value in describing the investor's utility function, then a natural consideration is the subset of (single-period) mean-variance efficient portfolios that have attractive long term investment properties (see also Michaud, 1981). This subset of the single-period efficient frontier provides the investor with a wide spectrum of choices of portfolios that are both near and long term attractive, avoiding the unnecessary limitation imposed by considering only the single (often high risk) portfolio that maximizes the (very long term) expected geometric mean (see, McEnally, 1986 for such a point of view).

6 The important exceptions to this rule are when the portfolio has no variance or the return distribution is (precisely) lognormal. Such a result points out that, while the lognormal return distribution assumption may be convenient in many instances, it may also lead to results that are misleading and not easily generalizable.

7 While the term $(1-P)^{1/N}$ in (3) increases with time, the "pure" expected geometric mean (second term) decreases with time. For intermediate values of P, the expected geometric mean may have an inflection point when N is greater than one. For illustrative purposes, we assume that the expected geometric mean is either an always increasing or always decreasing bounded function of the length of the investment horizon.

8 See Michaud (1981, sect. 3).

9 Financial applications of the Monte Carlo simulation technique are discussed in Lorie and Hamilton (1973, Ch. 15) and numerous other finance textbooks.

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FIGURE 1
COMPARISON OF EXPECTED PORTFOLIO GROWTH RATES
ORIGINAL VS TARGET PORTFOLIO
AVERAGE MARKET

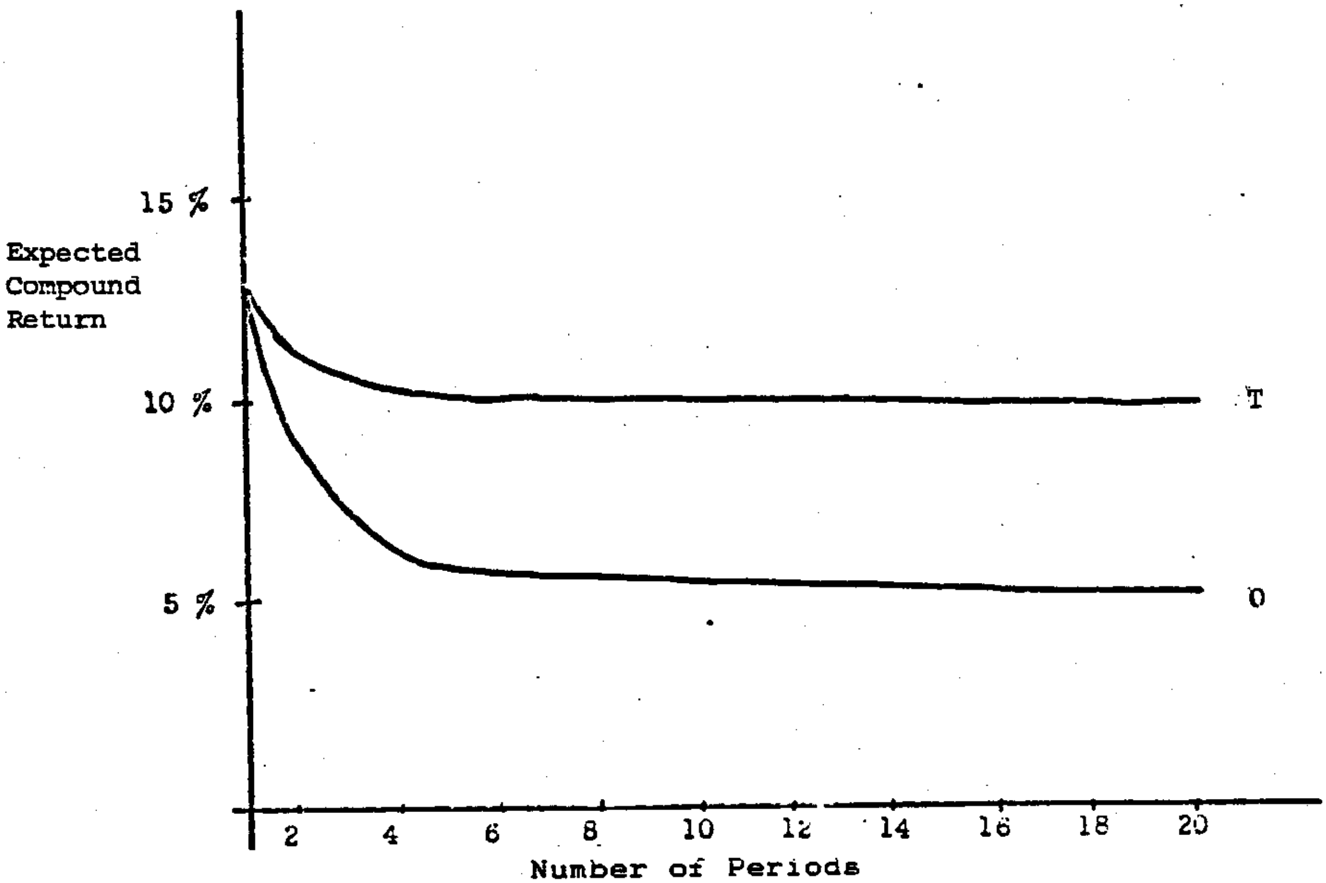

FIGURE 2
COMPARISON OF EXPECTED PORTFOLIO GROWTH RATES
ORIGINAL VS TARGET PORTFOLIO
ONE YEAR DIVERSIFICATION PROGRAM
AVERAGE MARKET

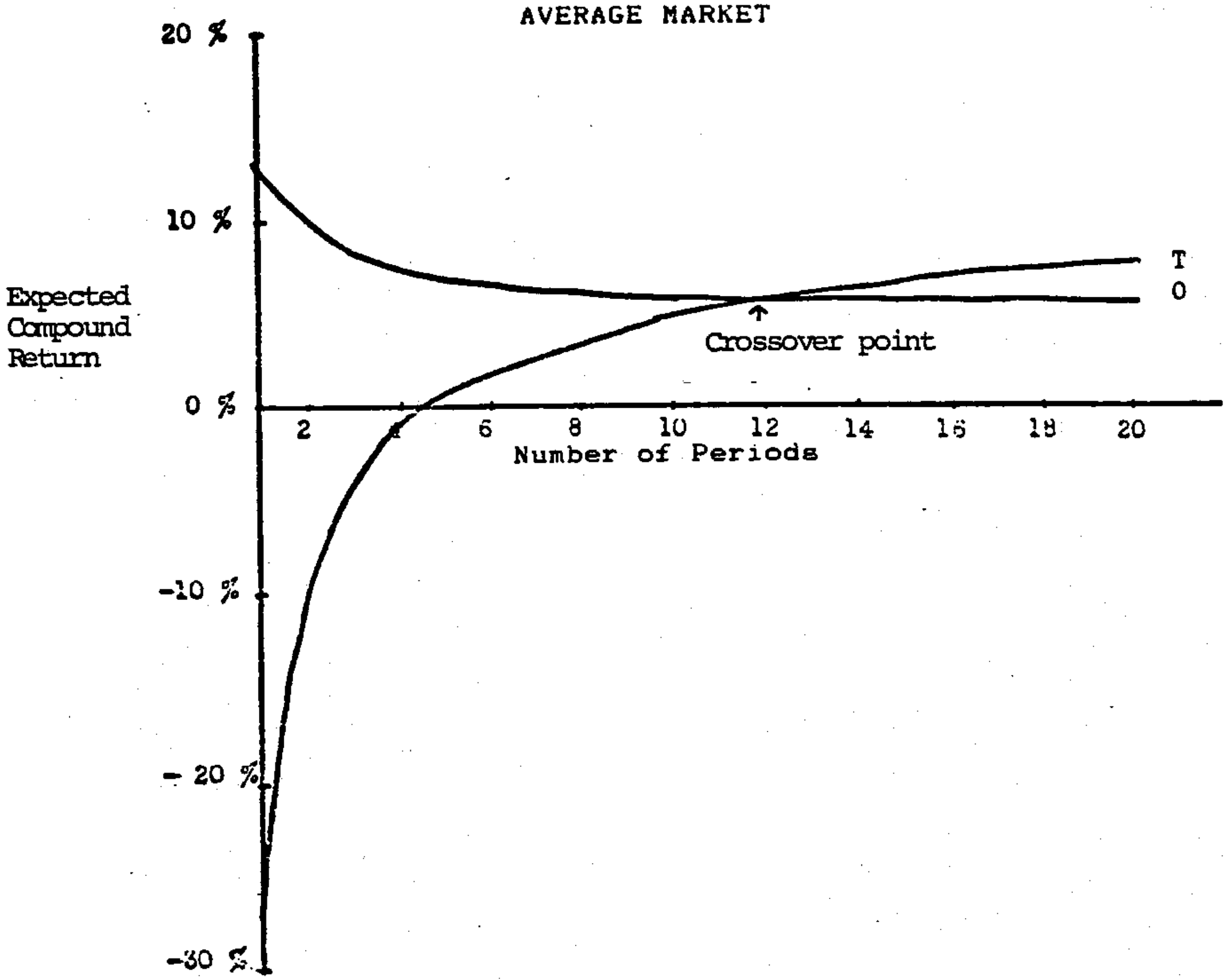

FIGURE 3
COMPARISON OF EXPECTED PORTFOLIO GROWTH RATES
BY INDICATED PERCENT OF ASSETS DIVERSIFIED
ONE YEAR DIVERSIFICATION PROGRAM
AVERAGE MARKET

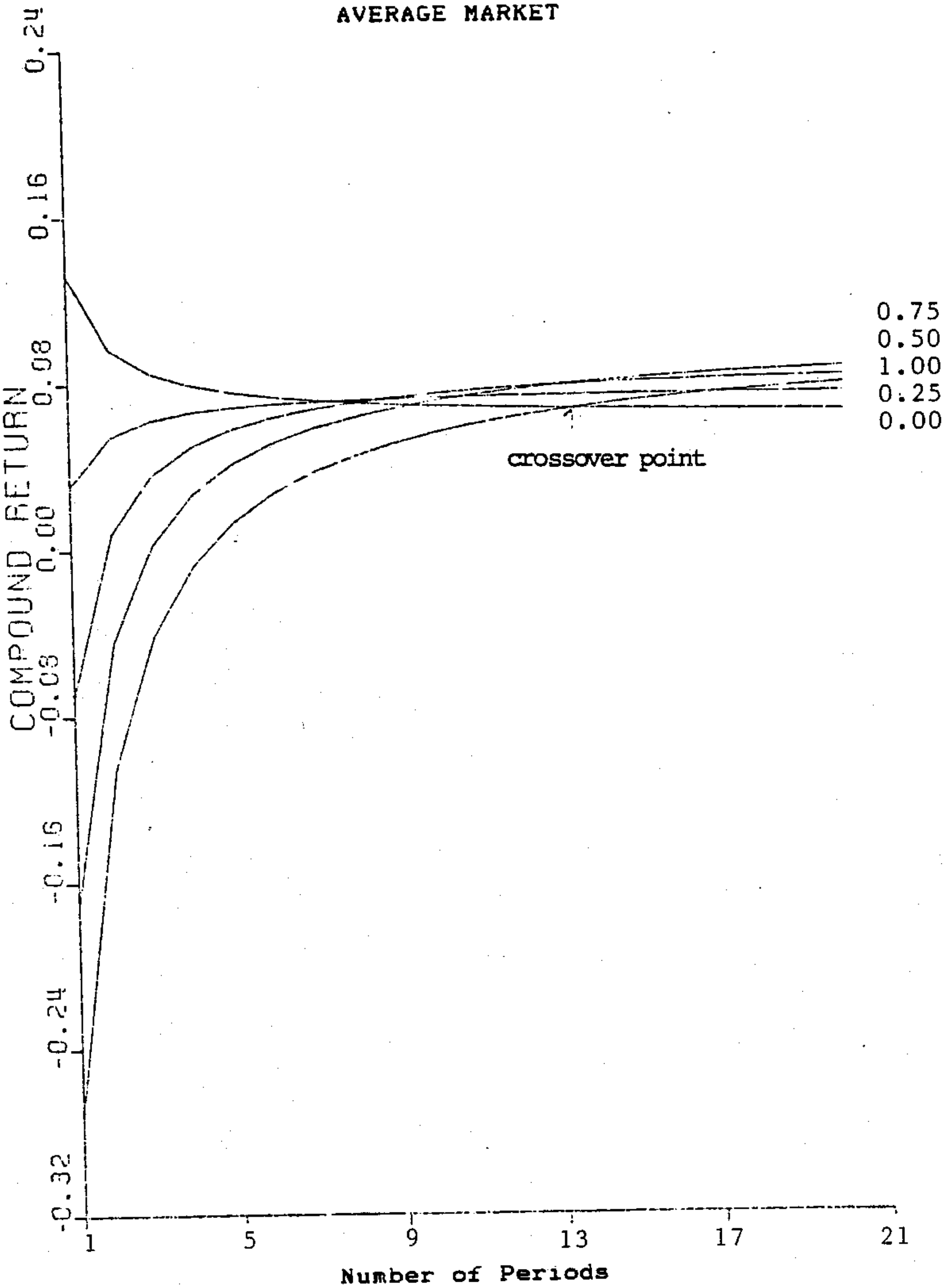

FIGURE 4
MEDIAN GROWTH RATE
BY INDICATED PERCENT OF ASSETS DIVERSIFIED
ONE YEAR DIVERSIFICATION PROGRAM
AVERAGE MARKET

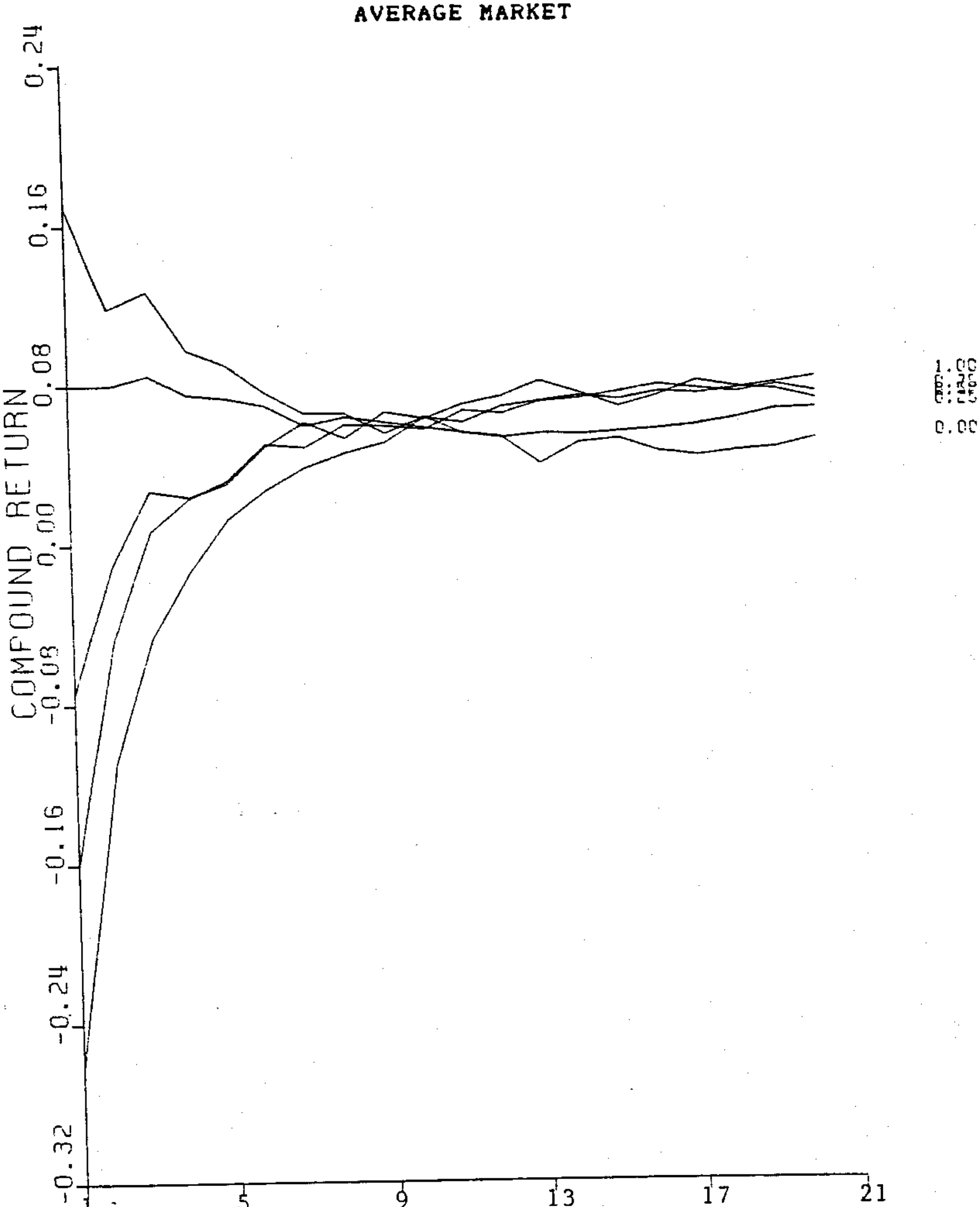

FIGURE 5
MEDIAN GROWTH RATE
BY INDICATED PERCENT OF ASSETS DIVERSIFIED
ONE YEAR DIVERSIFICATION PROGRAM
VOLATILE MARKET

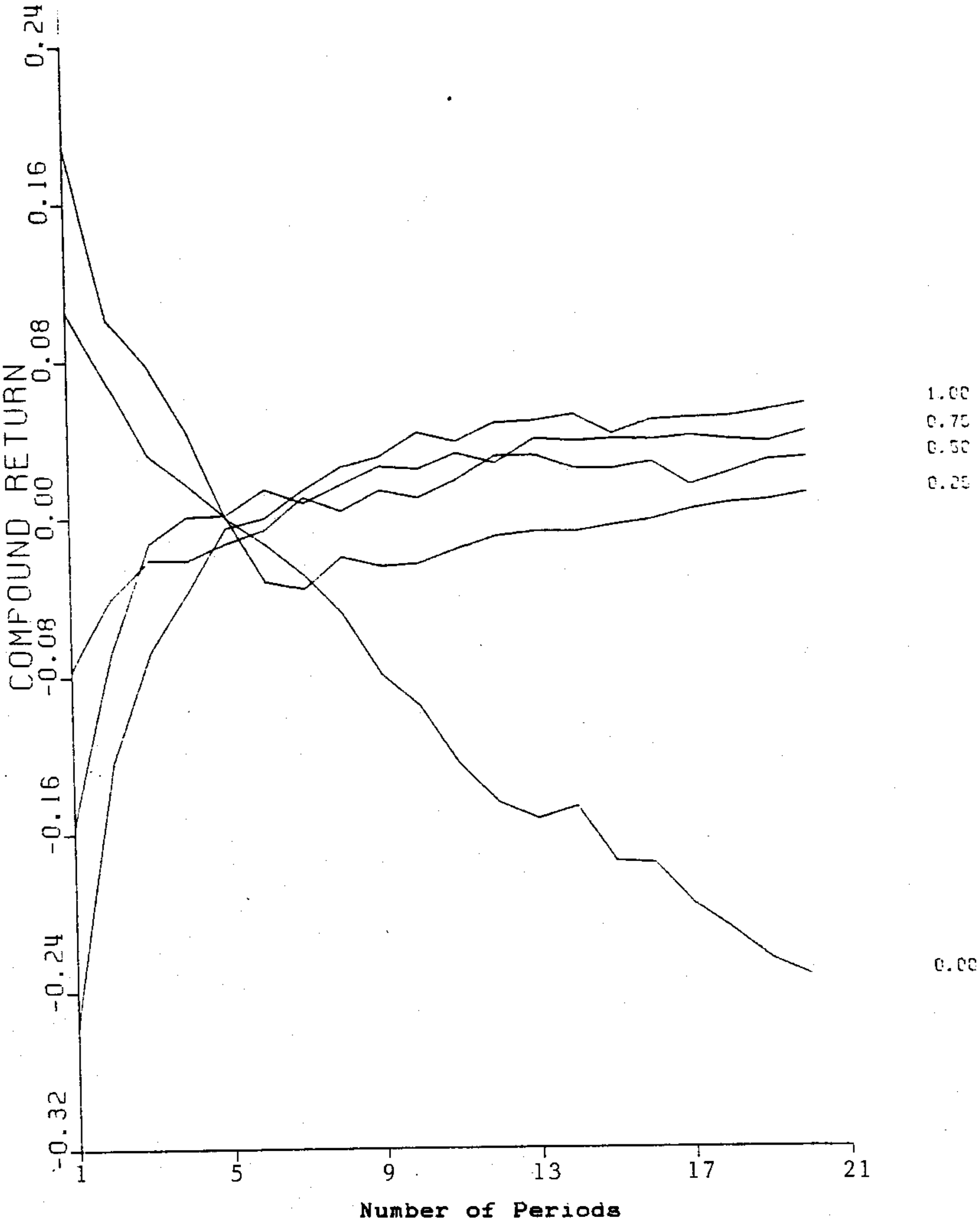

FIGURE 6
MEDIAN GROWTH RATE
BY INDICATED PERCENT OF ASSETS DIVERSIFIED
TEN YEAR DIVERSIFICATION PROGRAM
AVERAGE MARKET

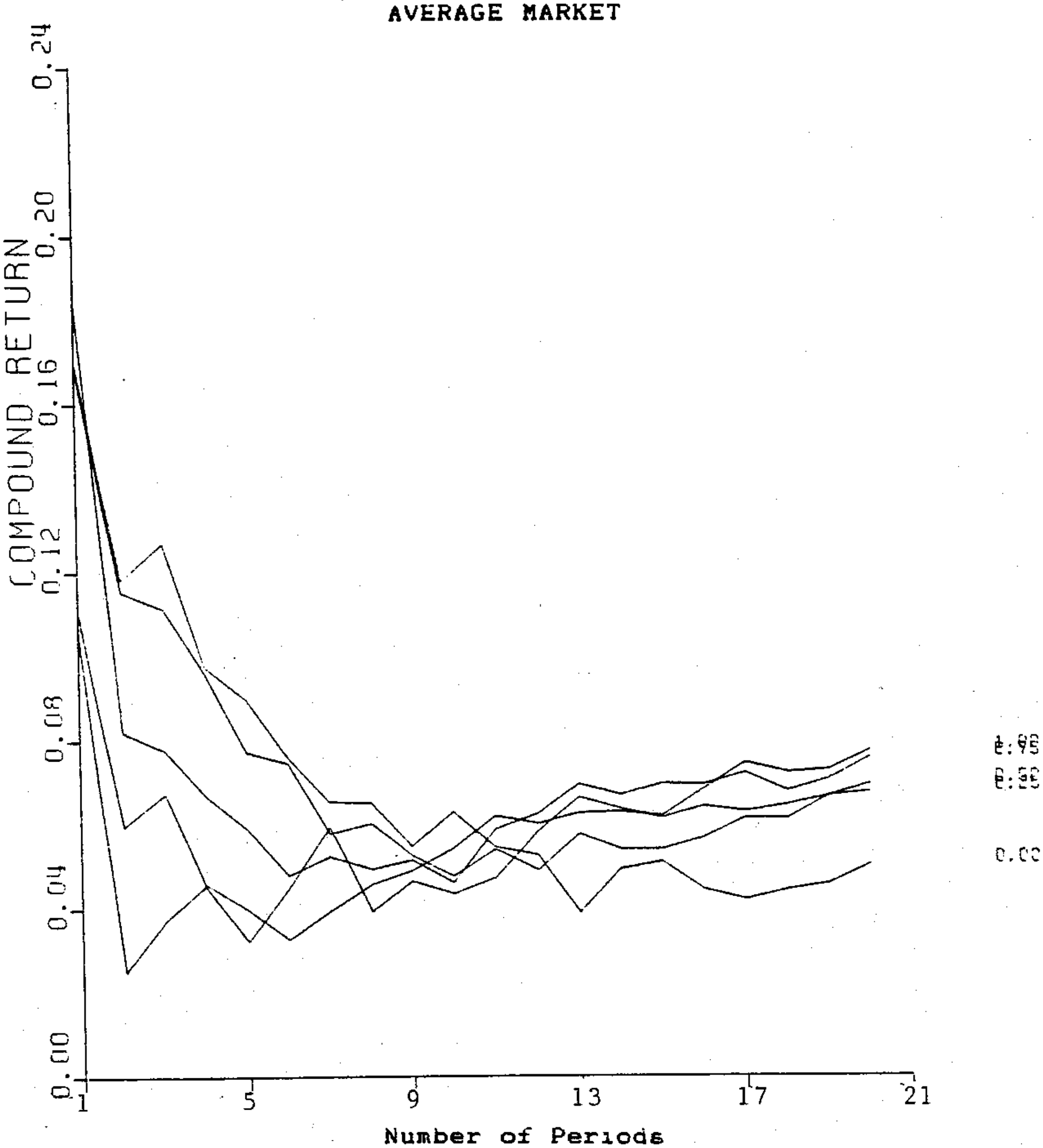

FIGURE 7
MEDIAN GROWTH RATE
BY INDICATED PERCENT OF ASSETS DIVERSIFIED
TEN YEAR DIVERSIFICATION PROGRAM
VOLATILE MARKET

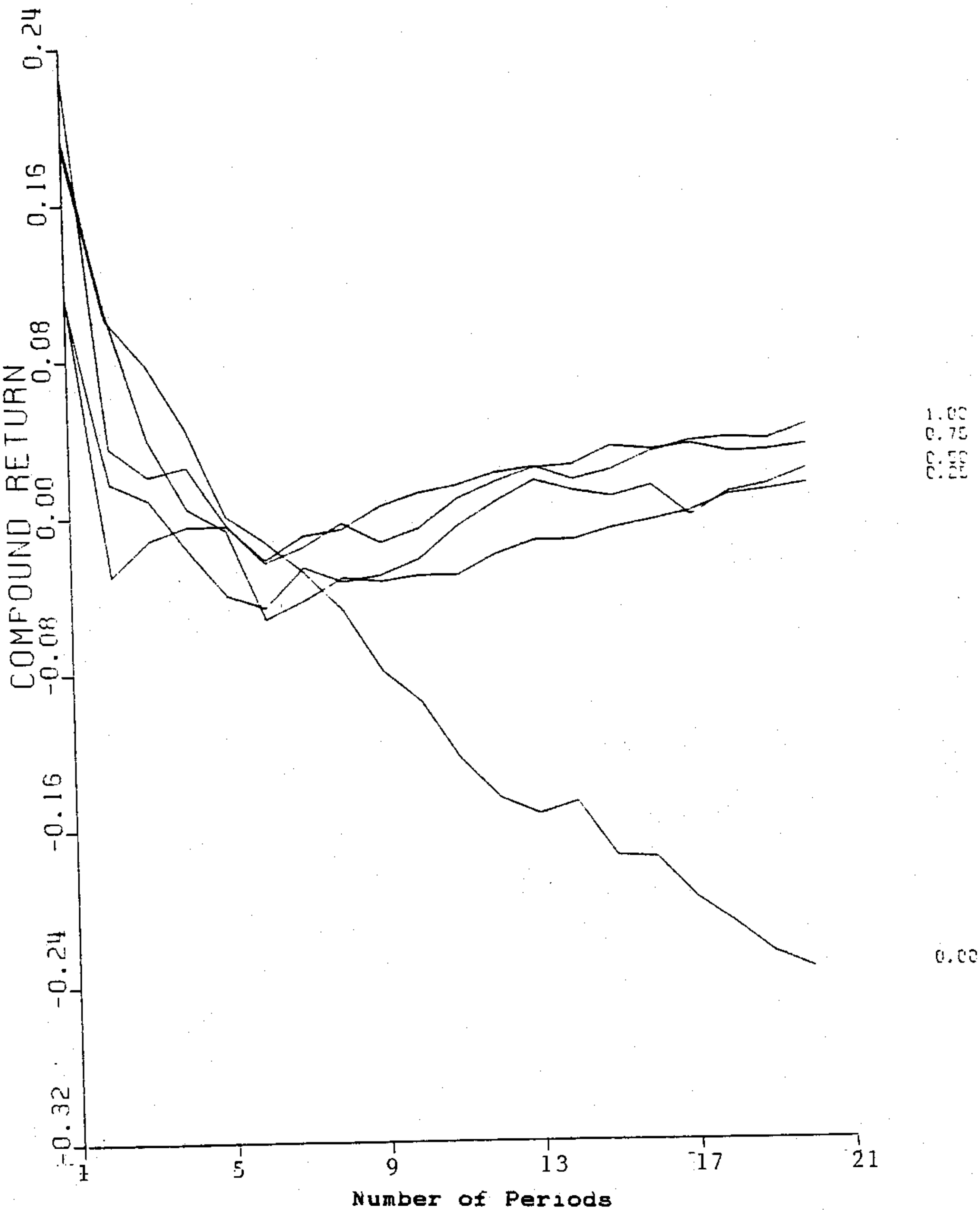

FIGURE 8
COMPOUND RETURN DISTRIBUTION
5TH YEAR
TEN YEAR DIVERSIFICATION PROGRAM
AVERAGE MARKET

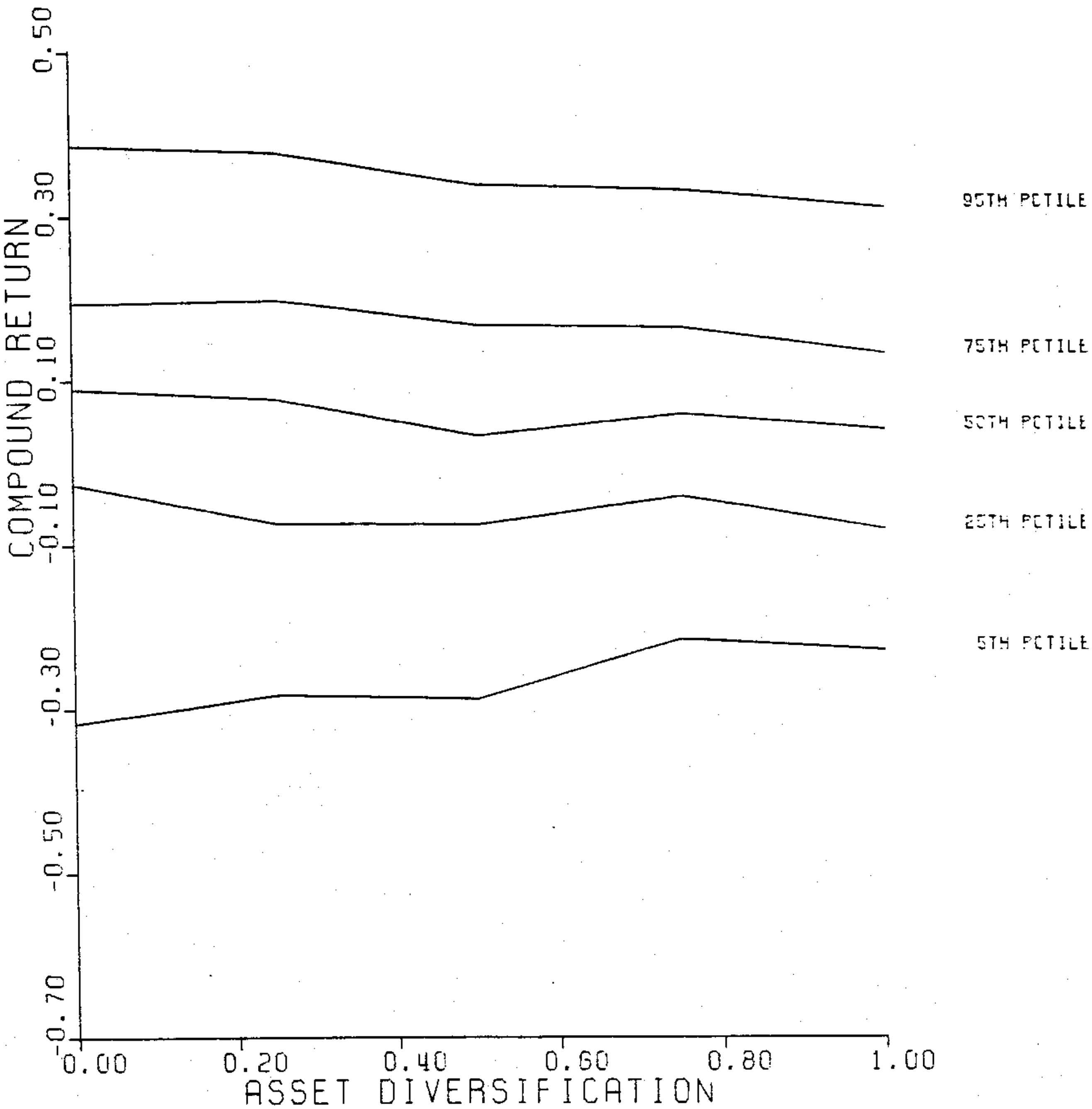

FIGURE 9
COMPOUND RETURN DISTRIBUTION
20TH YEAR
TEN YEAR DIVERSIFICATION PROGRAM
AVERAGE MARKET