An Introduction to Resampled Efficiency

By Richard O. Michaud

The proper purpose of investment advice is to improve a client's portfolio in terms of maximizing return for an appropriate level of risk. Asset management techniques for optimizing the investment value of forecasts of return and risk have been available for fifty years. Markowitz provides the classic definition of optimality: a portfolio is risk-return efficient if no other portfolio has higher expected return for a given level of risk or less risk for a given level of expected return.[1] The set of all portfolios that are risk-return efficient are said to form the Markowitz efficient frontier. Markowitz also developed mathematical methods for solving the risk-return optimization problem.[2]

From the perspective of fifty years, it is paradoxical that many managers and advisors don't use portfolio optimization investment technology in their work. While until relatively recently optimization computer programs were difficult to use and often required special analytical ability, it is not true today. Many commercially available programs for computing Markowitz efficient portfolios require virtually no computer or analytical expertise. Yet the paradox of managers and advisors ignoring investment technology that, conceptually, uses their information optimally remains. Are advisors not acting in their own or their clients' best interests?

Advisors offer a variety of excuses for not using portfolio optimizers. Some note lack of sophistication by their competitors or clients; others indicate that issues such as taxes or client servicing have priority over risk-return optimality. However, consider what would happen if it became widely known that advisors who use portfolio optimizers provide superior investment performance. Would a good client servicing bedside manner really compete with a competitor's ability to provide superior investment performance?

Consider an alternative hypothesis: portfolio optimizers have little, if any, investment value? If true, the benign neglect of modern portfolio theory by much of the investment community, however inconsistent with the exhortations of finance textbook writers, can be rationalized.

Optimizers have been found to have a number of serious limitations in investment practice. Many sophisticated asset managers and advisors have tried to use portfolio optimization technology in their business, and ended up discarding it. One reason is that optimizers are hard to manage. Even small changes in optimization inputs often lead to large changes in optimized portfolios. Since investment information is never known with certainty, the instability of the procedure leaves users with little confidence in the results. Advisors often include many constraints in order to stabilize the optimization. The end result, however, is that you constrain the problem to be what you wanted, and it is unclear what, if anything, the optimizer contributes to portfolio investment value.

Investment intuition can be a useful guide but it should be supplemented with scientific evidence if available. In the case of the investment behavior of portfolio optimizers, the evidence came more than twenty years ago, in a series of papers authored by two financial economists, J. D. Jobson and Bob Korkie, of the University of Alberta. They showed mathematically and statistically that optimizers have, on average, little, if any, investment value and that equal weighted portfolios are often far superior to optimized portfolios.[3]

Jobson and Korkie's papers provide the key to understanding the limitations of portfolio optimizers in practice. Classical optimizers assume 100% certainty in the information. But investment information is inherently uncertain. Optimizers tend to "error-maximize" investment information creating portfolio that are far too specific to a given s inputs and consequently reflect littl tuition or investment value.[4]

Resampled efficiency[5] provides solution to using uncertain informa in portfolio optimization. The meth based on resampling optimization i puts. This is a Monte Carlo simulat procedure to create alternative opti tion inputs that are consistent with uncertainty in your forecasts. To ill trate, suppose you forecast a 10% r and a 20% standard deviation for tl S&P 500 index. You would probat not be surprised to know that the tr return is 13% or 8% or that the true standard deviation is 25% or 15%. have, of course, uncertainty in the f casts of other assets as well. While mate uncertainty may be relatively to manage with one or two assets, i comes all but impossible when the folio has five or more assets or fun

The resampling process uses yo forecasts to indicate the many way capital markets and assets may beh in the investment period. This idea very similar to observing the behav of a fair coin when tossed ten time You expect to see five heads in ten es on average. But actual tosses of coin may result in two or nine hea even when the coin is fair. Simulat can show you how many ways the number of heads can occur and sin ly the many ways assets and capita markets can behave. There are mar ternative likely optimization inputs cient frontiers, and optimized portf consistent with your information. alternatives are critically important defining investment meaningful op mized portfolios.

But resampled efficiency is not ply a way of creating consistent al tive efficient frontiers and optimiz portfolios. Resampled efficiency is averaging process that distills all tl ternative efficient frontiers into a r efficient frontier and set of optimized portfolios. Intuitively, resampled efficient portfolios are optimal with respect to the many ways assets and capital markets can perform in the investment period consistent with your forecasts.

The historical risk-return data and optimized portfolio weights for the six indicated asset classes in the table below provide a simple comparison of classical and resampled optimized portfolios near the middle of the frontier and representing moderate risk. Note that, in this case, the classical portfolio has no allocation to small-cap stocks. This is because the risk-return estimates are used literally, and small caps have inferior estimates of return relative to large caps. In contrast, resampled efficiency includes a prominent component of small-cap stocks. This is because resampled efficiency is sensitive to uncertainty and uses investment information in a more robust manner. While the two portfolios are comparable, the resampled efficient portfolio is better diversified and intuitively less risky. This result is general.

Resampled efficiency is very simply a better way to use your investment information. It also has many investment attractive properties. Because resampled efficiency is an averaging process, it is very stable. Small changes in the inputs are generally associated with only small changes in the optimized portfolios. Resampled optimal portfolios are typically very investment intuitive without the need for constraints. This is because the uncertainty in your information, which is ignored with classical optimization, is being considered when defining the resampled optimized portfolio. Intuitive marketable optimized portfolios have the additional benefit of being a productivity tool, since few, if any, constraints are required to find suitable useful portfolios. Most importantly, resampled efficiency can be shown to improve investment performance on average.

Resampled efficiency is a new, more practical, definition of portfolio optimality. It is also a rigorous new patented tool for portfolio monitoring and rebalancing. This additional aspect of resampled efficiency is described further below.

Each portfolio on the resampled efficient frontier is the result of averaging a number of statistically equivalent efficient portfolios. This creates what may be described as a fuzzy set of statistically equivalent portfolios associated with each portfolio on the resampled efficient frontier. For a given resampled efficient frontier portfolio, the fuzzy region of associated portfolios, in its simplest form, can be visualized as a set of points filling the inside of an American football with the resampled efficient portfolio in the center. Now think about comparing your portfolio to a resampled efficient frontier portfolio. Is your portfolio inside the football and close to the center or far from the center or even outside the football? If it's far from the center, it's not statistically similar and you may want to consider rebalancing it. On the other hand, if your portfolio is close to the center, you may not need to rebalance the portfolio at all because it's statistically similar. The rule computes a need-to-trade probability. A probability of 95% indicates that trading appears advisable; alternatively, a 20% probability may indicate no need to rebalance. Because of the nature of the portfolio rebalancing problem, intuition is typically an unreliable guide. A portfolio may look the same as another and perform very differently and conversely. In many cases, reliable decisions require rigorous statistical analysis.

The resampled trading rule is the first statistically rigorous portfolio rebalancing rule available to the investment community. Avoiding trading without benefit enhances investment performance independent of the optimization process. The trading rule is also a highly scalable and automatable portfolio monitoring procedure. For a given resampled efficient frontier, hundreds of portfolios can be monitored with common laptop technology in a very short period of time. It is also easy to customize the monitoring process to accommodate client trading frequency mandates, changes in the certainty of investment information, and investment strategy trading patterns.

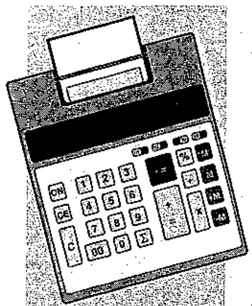
As it turns out, investment professionals were well advised not to use commercial portfolio optimizers because they did not take forecast uncertainty into consideration. However, resampled efficiency, which explicitly considers investment information uncertainty, is a new class of portfolio optimization technology that holds out the promise of nearly automatable, customizable and effective asset management as well as new asset management tools not previously available. Thoughtful asset managers and advisors may wish to reconsider their aversion to portfolio optimization tools. IMCA

Classical vs. Resampled Optimal Portfolios[7]

Assets	Exp. Retn (%)	Std. Dev (%)	Classical (%)	Resampled (%)
T-bills	4.6	0.4	0	6
Interm Govt	7.1	4.3	36	18
LT Corp	8.3	6.4	0	12
LC Stock	17.7	13.4	64	54
SC Stock	15.7	17.4	0	10
Intern Stock	8.3	17.1	0	0

Endnotes

1. Harry Markowitz, 1952. "Portfolio Selection." *Journal of Finance* 7(1): 47-62.
2. Harry Markowitz, 1956. "The Optimization of a Quadratic Function Subject to Linear Constraints." *Naval Research Logistics Quarterly* 3(1/2): 111-133.
3. J.D. Jobson and Bob Korkie, 1981. "Putting Markowitz Theory to Work." *Journal of Portfolio Management* 7(4): 70-74.
4. R. Michaud, 1989. "The Markowitz Optimization Enigma: Are Optimized Portfolios Optimal?" *Financial Analysts Journal.*
5. Richard Michaud, 1998. *Efficient Asset Management.* Oxford University Press, 1991. Originally published by Harvard Business School Press.
6. The first two columns in the table display the annualized means and standard deviations of monthly data for six asset classes for ten years. The last two columns display the classical and resampled optimal portfolio weights with equal annualized standard deviations of 9% in the middle of their respective frontiers.



Investment Benchmark Scorecard

CALCULATING RETURNS

PERIODS ENDING DECEMBER 31, 2002

	Return Last Quarter	Return Last Year	Return Five Years	Risk Five Years	Return Ten Years	Risk Ten Years	Sortino Ratio T-Bills Five Years	Sortino Ratio S&P 500 Five Years
Equity Indices								
Dow Industrials	10.62%	-14.99%	0.99%	18.52%	11.06%	15.67%	(0.25)	0.25
S&P 500 Industrials–Cap Weighted	8.44%	-22.09%	-0.58%	18.74%	9.35%	15.44%	(0.35)	0.00
S&P 500 Industrials–Eql Weighted	13.04%	-17.52%	3.45%	20.15%	11.17%	16.08%	(0.07)	0.80
Domini Social	9.24%	-20.10%	0.17%	19.83%	9.99%	16.29%	(0.29)	0.30
NYSE	6.70%	-18.40%	-0.19%	16.33%	8.99%	13.74%	(0.37)	0.10
AMEX	0.00%	-1.57%	5.08%	17.78%	8.65%	14.93%	0.04	0.60
Russell Top 200	9.48%	-22.49%	-1.43%	19.44%	9.05%	15.97%	(0.40)	(0.48)
Russell 1000	8.16%	-21.65%	-0.58%	18.93%	9.18%	15.51%	(0.35)	(0.00)
Russell 2000	6.15%	-20.48%	-1.34%	23.81%	7.17%	19.07%	(0.32)	(0.06)
Russell 3000	8.02%	-21.55%	-0.72%	18.87%	8.93%	15.42%	(0.36)	(0.08)
Russell 1000 Growth	7.14%	-27.89%	-3.84%	24.56%	6.71%	19.48%	(0.44)	(0.45)
Russell 1000 Value	9.22%	-15.53%	1.16%	17.19%	10.81%	14.23%	(0.26)	0.27
Russell MidCap	7.91%	-16.19%	2.19%	19.49%	9.92%	15.77%	(0.16)	0.53
Wilshire 5000	7.82%	-20.85%	-0.86%	19.24%	8.75%	15.62%	(0.36)	(0.10)
Wilshire Small Growth	4.81%	-12.71%	-0.86%	24.07%	7.33%	20.62%	(0.29)	(0.03)
Wilshire Small Value	5.94%	-7.63%	-0.11%	16.69%	8.24%	13.38%	(0.34)	0.04
NASDAQ	14.11%	-31.13%	-2.63%	37.36%	8.05%	28.44%	(0.26)	(0.12)
EAFE	6.48%	-15.64%	-2.60%	16.92%	4.31%	15.46%	(0.52)	(0.25)
Financial Times–World	7.97%	-19.01%	-1.37%	17.43%	6.58%	14.64%	(0.43)	(0.22)
Financial Times–Pacific	-1.86%	-9.16%	-2.82%	20.62%	-1.10%	20.43%	(0.50)	(0.19)
Financial Times–Europe	11.28%	-17.39%	-1.98%	17.95%	8.19%	15.10%	(0.45)	(0.16)
Debt Indices								
ML Domestic Master	1.59%	10.41%	7.59%	3.35%	7.57%	3.73%	1.53	0.65
ML Corp/Gov't Master	1.61%	11.00%	7.62%	3.90%	7.63%	4.17%	1.30	0.64
ML Intermediate Corp/Gov't	1.61%	9.65%	7.46%	2.94%	7.09%	3.04%	1.78	0.64
ML Mortgage Master	1.54%	9.42%	7.48%	2.51%	7.40%	2.96%	2.11	0.65
ML High Yield Master	6.68%	-1.13%	0.91%	8.12%	6.15%	6.47%	(0.55)	0.12
ML Long Treasuries	0.00%	16.76%	8.70%	7.72%	9.37%	8.09%	0.84	0.64
ML Int'm Treasuries	0.73%	9.07%	7.26%	2.95%	6.92%	2.95%	1.64	0.62
ML 1–3 Yr Treasuries	0.93%	5.81%	6.42%	1.56%	6.05%	1.62%	2.50	0.57
ML Municipal Master	-0.45%	10.72%	6.34%	5.41%	6.85%	5.31%	0.52	0.50
Ryan Labs 3 mo. T-Bills	0.41%	1.68%	4.49%	0.51%	4.72%	0.43%	0.00	0.41
Ryan Labs 6 mo. T-Bills	0.44%	1.96%	4.54%	0.56%	4.81%	0.50%	0.21	0.42
Ryan Labs 2 Yr. Treasury	0.90%	6.88%	6.06%	1.93%	5.92%	1.89%	1.45	0.53
Ryan Labs 5 Yr. Treasury	0.41%	12.53%	7.52%	4.55%	7.15%	4.40%	1.10	0.61
Ryan Labs 10 Yr. Treasury	-0.41%	15.80%	7.40%	6.92%	7.46%	6.82%	0.66	0.56
Ryan Labs 30 Yr. Treasury	-0.64%	16.18%	7.75%	10.09%	8.52%	10.18%	0.50	0.54
Real Estate Indices								
Nat'l Assoc. of REIT's–Equity	0.41%	3.81%	3.30%	12.99%	10.53%	12.31%	(0.13)	0.28
Nat'l Assoc. of REIT's–Hybrid	4.36%	23.30%	-2.58%	21.05%	6.90%	16.51%	(0.44)	(0.10)
Nat'l Assoc. of REIT's–Mortgage	10.21%	31.09%	4.97%	25.16%	10.96%	20.68%	0.02	0.27
World Assets								
Brinson Multiple Markets	5.48%	-6.65%	3.04%	10.15%	7.97%	8.48%	(0.18)	0.57

Return = Total Returns, annualized, except for the latest quarter Risk = Annualized Standard Deviation

Data provided courtesy of Frank Russell Co., National Association of REIT's, Ryan Labs, Merrill Lynch, Wilshire Associates, Brinson Partners, and Kinder, Lydenberg, Domini, Inc.

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