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# The Right Tool for Picking an Investment Vehicle: The Geometric Mean

by Dr. David Esch

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Let's assume that you want an excellent asset allocation, one designed to take on a particular level of risk while providing an expectation that the return will exceed returns generated without that risk. Michaud optimization, applied to a comprehensive global universe of funds providing exposure to multiple risk premia, gives a set of portfolios that generate the best forward-looking expected return while suitably controlling risk. Although the Michaud algorithm creates the best portfolios for long-term investing, it does not completely solve the investment problem, because the investor must still decide which portfolio is most consistent with his or her goals and risk appetite. Small changes in the contribution amount, risk level, investment horizon, or liability requirements can drastically affect the chances of attaining the goal of funding liabilities. All of these moving parts affect the choice of which portfolio is best.

One unexpected consequence of long-term investing is that risk must be considered much more carefully than for short-term investing. For example, if there is only a 5% chance of a return less than a certain threshold (a catastrophic loss) in any particular calendar year, then there is a 78.5% chance of this catastrophic event happening sometime over a period of thirty years. Another factor that must be considered is that downside risk is more destructive to wealth than arithmetically equivalent upside risk. At the extreme, a –100% return leaves the investor with zero wealth, where the "equivalent" upside 100% return doubles the investment. +100% is clearly not equivalent to -100% for returns; rather it is equivalent to -50%, since wealth is growing or shrinking by the equivalent factor of 2. When returns are small, say, between -5% and +5%, the effect of adding versus compounding multiplicatively are small, but over a long investment period, where the total return is on the order of doubling the investment or more, this effect becomes crucial. A 5% return over 20 years is just 100% when added together, but the "geometric" calculation is ((1.05)20-1)*100% = 165.33%, a discrepancy of 65%. To then calculate the average return, rather than dividing the return by the horizon, averages are calculated exponentially, taking the nth root, where n is the number of years in the horizon. This geometric mean is a better approximation of the probable long-term return than the arithmetic mean.

The geometric mean becomes more complicated when the returns are not known, but rather characterized by a random probability distribution. If each year has a 5%


Dr. David Esch

Executive Director of Research
New Frontier


Dr. David Esch is the Managing Director of Research at New Frontier, having joined the firm in 2008. Dr. Esch completed his Ph.D. in Statistics at Harvard University in 2004. His specialties include mathematical statistics, numerical analysis and computation, Bayesian statistics, and econometrics. He is author of the article "Non Normality Facts and Fallacies," (Journal Of Investment Management 1st Quarter 2010), selected as one of the best JOIM papers of 2010, and co-author of many other peer-reviewed journal articles. His educational background also includes a Bachelor of Arts degree from Harvard College and a Masters degree in Mathematics and Statistics from Boston University.

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expected return, but these returns are risky and fall above or below the 5% mark on any given year, the expected geometric mean over the 20 year investment horizon will be less than the 165% total return calculated above when the 5% returns are certain. The greater the risk, the greater the penalty paid in the expected terminal wealth. It is because of this potential of downside risk to erode the chances of goal attainment that a sensible risk management plan such as using a portfolio from the risk-managed Michaud frontier is so crucial. Even with a risk-controlled product, the entire time horizon of the investment must be examined to appreciate the possible consequences of taking more risk than necessary, and the geometric mean is the appropriate tool to start with.

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