Resampled Portfolio Rebalancing and Monitoring

by

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Abstract

This paper describes an enhancement to the original rebalancing procedure that dramatically increases the uniformity and discrimination power of the original portfolio rebalancing and asset weight range procedures.

A statistically rigorous method for portfolio rebalancing and monitoring is presented in Michaud (1998, Ch. 7).[1] This procedure, based on a patented resampled optimization technology, was the world's first rigorous portfolio trading rule and asset weight range estimator.[2] An enhancement to the original procedure is now available that dramatically increases the uniformity and discrimination power of the original portfolio rebalancing and asset weight range procedures.[3]

Past and Present Portfolio Rebalancing Rules

Up until recently, portfolio rebalancing rules were ad hoc. Two kinds of rebalancing rules characterize much of the investment community's practice: 1) Calendar rules for monthly, quarterly or annual rebalancing; 2) Trading if an asset weight lies outside a plus or minus range of 5 or 10% relative to the optimal portfolio's weights. A less common but still fallacious approach is to rebalance a portfolio based on its nearness to an optimal portfolio in terms of expected return and risk.[4]

A valid trading rule has to be based on portfolio structure and portfolio characteristics as a whole. For example, if two assets are near substitutes, an investor will be indifferent to a 10% weight in either or 5% in both.[5] Up until recently, no statistically rigorous portfolio structure based rebalancing rule has been available to the investment community. Yet rebalancing decisions are among the most important activities of an effective asset manager. Monitoring of a portfolios' optimality is often a full time activity for many financial advisors, portfolio managers, and trust officers. Too much, as well as too little, trading may substantially reduce the value and effectiveness of investment information and result in suboptimal portfolio performance.

The Resampled Portfolio Rebalancing Rule

The resampled portfolio rebalancing rule is based on a patented portfolio optimization process called resampled efficiency. The resampling process, properly applied, leads to a new mean-variance efficient frontier that is provably effective at enhancing investment value. The new resampled efficient frontier is also investment intuitive, easy to manage without the need of ad hoc constraints, and robust with respect to changes in optimization inputs.[6] The patented resampling process also creates a statistical distribution of portfolio structure distances for each portfolio on the resampled efficient

[1] Michaud, Richard, 1998. *Efficient Asset Management*. Oxford University Press, New York.

[2] Richard Michaud and Robert Michaud co-inventors, December 1999, U.S. Patent # 6,003,018.

[3] Patent pending.

[4] Utility based methods are a generalized case of the same fallacy. This is because the utility function summarizes all the information about the portfolio in terms of its expected return and variance (and possibly other moments). This is equivalent to the "sufficient statistic" concept in mathematical statistics. However, in this case it is an invalid application of the statistical method. There is also the critically important additional limitation of using a utility function to define nearness since any choice implies risk averseness characteristics that are unlikely to be appropriate for a very wide spectrum of investors.

[5] The two assets are substitutes in the sense of similar risk, expected return and correlations with other assets.

[6] See New Frontier Advisors, LLC October 2002 Newsletter for a simple review and Michaud (1998, Ch. 6) for a more detailed description.

frontier. This distribution provides a way to rigorously measure at a given confidence level whether a portfolio is near or far from optimality. The obvious advantages of the resampled portfolio rebalancing rule include automatable and objective portfolio monitoring while limiting the need to trade without benefit. It is a highly scalable procedure that is easy to use and understand. The resampled rebalancing methods also provide a statistically rigorous estimate of efficient portfolio confidence ranges for each asset. Such information is useful in practice to determine at-a-glace indications of the sources of asset weight imbalances in a non-optimal portfolio.

A Meta-Resampled Portfolio Rebalancing Rule

Our recent research in statistical optimization has led to a new method that substantially enhances the investment value of estimates of the need-to-trade probability and asset weight confidence ranges. Our improved estimates use a meta-resampling technique to find more appropriate portfolios in the computation of the portfolio distance distribution.[7] Meta-resampling leads to a dramatic increase in the discriminatory as well as uniformity of the power of the resampled rebalancing test over the entire range of the resampled efficient frontier. In the original version of the resampled rebalancing rule, the discriminatory power was too high at low risk efficient portfolios and too low at high risk. This is no longer the case. In addition, there is a concomitant increase in the accuracy and investment usefulness of confidence range estimates for all portfolios on the resampled efficient frontier.

An Illustration

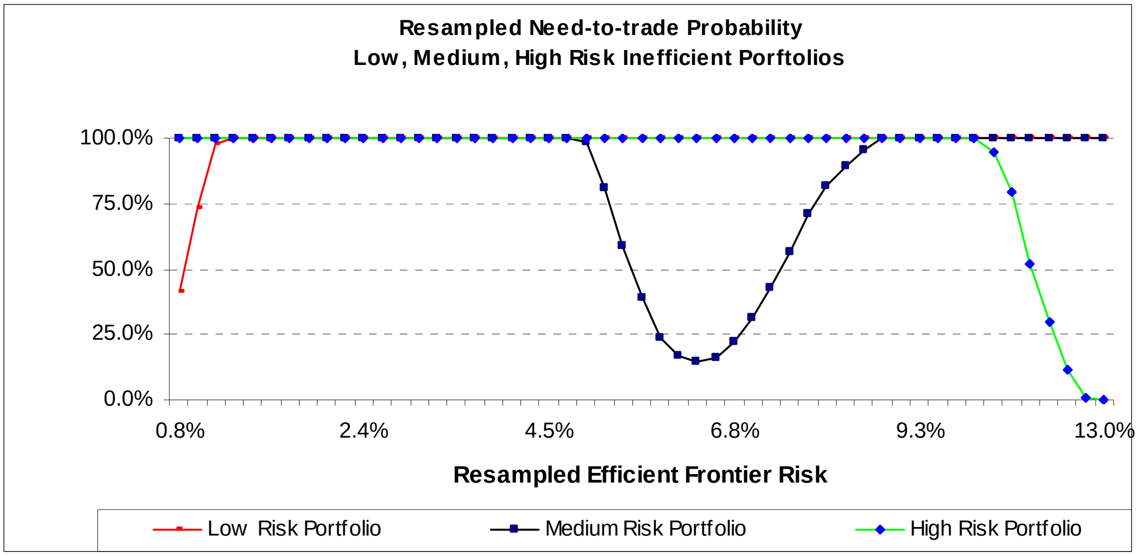
The accompanying exhibit illustrates the power and investment characteristics of the new rebalancing technique. Three resampled inefficient portfolios were chosen that lie near the resampled efficient frontier at minimum risk, middle, and maximum expected return. The exhibit displays the new rebalancing when-to-trade probability for each of the three inefficient portfolios relative to every portfolio on the resampled efficient frontier. The when-to-trade probability graphs are similar to an inverted bell curve where the low point is at the risk level of the resampled optimal portfolio closest in portfolio distance to the given inefficient portfolio. More specifically, consider the middle inefficient portfolio which has roughly 6% risk. As the exhibit shows, the when-to-trade graph has its low point at roughly 6% portfolio risk and increases nearly symmetrically around this point until it reaches 100% probability values for efficient portfolios some distance from the middle portfolio.[8] In the case of the minimum risk and maximum expected return portfolios, the when-to-trade probability graphs are necessarily one-sided.

One of the most important features of the new rebalancing rule is that it is reliable not only for indicating if a trade is likely to be ineffective but also when trading is advisable. To see this note how quickly the rebalancing probabilities increase even though the

[7] Very briefly, meta-resampling techniques compute resampled efficient frontiers from resampled optimization inputs that are then used in the new rebalancing procedures. We will be reporting on this new development in more depth in future newsletters and subsequent research reports.

[8] Note that the mild asymmetric character of the when-to-trade probability graph for the middle inefficient portfolio is consistent with the financial reality of increasing estimation error as a function of increasing risk.

inefficient portfolios are very close (from a mean-variance perspective) to portfolios on the resampled efficient frontier.



Resampled Rebalancing Methods

We have developed a fast-algorithm version of the meta-resampling need-to-trade probability and asset weight confidence ranges.[9] The new procedure takes virtually the same amount of computational time as current resampled portfolio optimization.[10] The fast-algorithm method is a recommended optimized balance between absolute accuracy and practical investment requirements for a wide range of applications and users.[11] However, in cases where accuracy is paramount and the optimization problems are reasonably small, a second compute-intensive option for meta-resampled portfolio rebalancing is available.[12] This second option is more accurate but substantially more computationally intensive. It is likely to be most useful for asset allocation studies. For compute-intensive meta-resampling, the need-to-trade probability and asset weight ranges may be exact to the limit of the number of simulations used. However, this option should be used with awareness of its practical consequences.

[9] The results in the exhibit are taken from the compute-efficient algorithm.
[10] Compute-efficient methods are extremely important in practice for certain applications such as institutional equity portfolio optimization. New Frontier Advisors, LLC compute-efficient meta-resampling portfolio rebalancing technology is currently available in Northfield Information Services equity portfolio optimizer.
[11] There is a small bias in the compute-efficient algorithm for overstating the need-to-trade probability and understating the asset weight ranges. Asset managers may wish to adjust their use of the procedure accordingly.
[12] New Frontier Advisors, LLC (NFA) meta-resampled portfolio rebalancing software is available for both compute-efficient and compute-intensive versions. NFA Excel desktop users have access to either as an option on the input sheet page of the ROM-optimizer module.