NEW FRONTIER ADVISORS, LLC

Equity Optimization Inputs

by

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Abstract

The first of five equity optimization articles presents a beginning discussion of difficulties faced by traditional optimizers and the solutions NFA began to explore.

Resampled Efficiency™ (RE) optimization[1] is a generalization of Markowitz mean-variance (MV) optimization that includes estimate uncertainty in the portfolio optimization process. RE optimization is a necessary condition for defining optimal risk-return portfolios. It is the only optimizer with a rigorous statistical proof of investment effectiveness in the world today. However, many practices commonly associated with traditional mean-variance (MV) equity optimization may limit the ability of RE optimization to improve investment value. This purpose of this newsletter is to set the stage for a series of discussions on specific issues related to equity portfolio optimization and best practices. In this note, we examine some often self-defeating practices associated with MV optimization and develop an RE equity optimization framework to avoid them.

Unoptimized Portfolios

Markowitz (1959) mean-variance (MV) efficiency has been the standard for defining portfolio optimality for nearly fifty years. The rationale for Markowitz efficiency is that it is a useful approximation to expected utility maximization for many problems in investment practice. However, Markowitz optimization is widely known to be highly unstable. If the inputs are not carefully managed or the optimizer not carefully constrained the optimized portfolio is rarely investable. More importantly, rigorous statistical tests confirm that the MV optimization process has, on average, no effective investment value. MV optimization in practice is useful primarily for marketing and factor structuring purposes and not for defining risk/return optimality.

A Statistical Objective

There is a great deal of statistical uncertainty in typical investment information. Markowitz optimization is a numerical optimization procedure that is, by definition, insensitive to statistical uncertainty in risk and return estimates. A numerical optimization procedure is useful for many purposes. But portfolio risk-return optimality is not one of them. A statistical optimization objective necessarily requires a statistical optimization procedure and MV efficiency fails in creating investment effective risk-return optimal portfolios.

A Statistical Optimization Solution

In order to properly optimize portfolios it is necessary to include estimate uncertainty in the optimization process. Using resampling methods, RE optimization is a generalization of MV efficiency that properly addresses statistical uncertainty in risk-return estimates. The RE optimizer leads to intuitive and marketable portfolios, often without the need for any ad hoc constraints, and is very stable. Most importantly, it is the only optimization framework with a rigorous proof of investment effectiveness in the world today.[2]

[1] Richard Michaud and Robert Michaud are co-inventors: U.S. Patent awarded December 1999, worldwide patents pending. New Frontier Advisors, LLC is the exclusive worldwide licensee.
[2] The interested reader is referred to Michaud (1998, Ch. 6, 7, Oxford University Press) and many newsletter articles on www.newfrontieradvisors.com.

Contrary to widespread investment tradition, recent evidence confirms that the critical factor in optimized portfolio performance is the RE optimizer not the inputs. [3]

Mental Retooling

RE optimization provides a framework with the potential for substantially improving risk-return portfolio optimality. But like any statistical procedure, RE optimization can be misused. The benefits of RE relative to MV optimization require mental retooling on the part of the investment community. Investors need to be aware of the information level in their estimates. It is important to introduce safeguards in the RE optimization framework in order to avoid the potential limitations of many common MV optimization practices in the investment community today. While mental retooling is important in all uses of MV optimization, it is particularly so for equity optimization. This is because investors are far less able to be guided by investment intuition when optimizing 500 or 7000 securities as when optimizing 10 asset classes in an asset allocation.

Avoiding Too Much Risk

The typical MV equity optimization objective is formulated in terms of tracking error or residual risk-return.[4] Many optimizations may include 500 or more securities. A commercial risk model is often used to define the components of security risk. Particularly for large stock universes, tracking error risk for many securities can be many multiples of market risk.

There are many commercial forces in the investment community today that encourage asset managers to assume more than ordinary amounts of portfolio tracking error risk. For example, hedge fund managers typically assume far higher levels of tracking error risk than traditional long-only institutional managers. Also, the currently topical core-satellite framework encourages fund trustees to reduce allocations and demand higher risk levels of active asset managers. Can an equity manager take too much risk?

As every good investor knows, one of the most critical investment decisions is assuming the correct amount of risk. Too much risk is imprudent and often commercially fatal. As the demand for portfolio risk increases, estimate uncertainty increases. At some point is it reasonable to assume that there may be too much risk demanded of an optimized portfolio?

In the world of MV optimization, no such warning exists. But this should not be surprising to the reader who has followed the arguments in this note. In one of the most significant contributions of RE optimization, the investor is able to determine if the level of risk desired is inconsistent with portfolio risk-return optimality. This gives rise to the concept of the Resampled Efficient Frontier™ (REF) "critical risk" point.

[3] Markowitz and Usmen (2003, *Journal of Investment Management*, 4th Quarter).
[4] Tracking error or residual return refers to return relative to a benchmark return or index. Tracking error return may or may not include factor and group bets as well as stock specific return.

The RE Frontier Critical Risk Point

As optimized portfolio tracking error risk is increased so is estimate uncertainty. At some point on the tracking error risk spectrum, the uncertainty in the estimates may begin to overwhelm the benefit of assuming more risk. In this case the RE frontier will reach a maximum estimated return and higher risk levels are risk-return inefficient. Essentially, the investor is demanding too much risk for the level of information available in the estimates. It is only because MV optimization is insensitive to estimate uncertainty that the critical point has never been observed. The investor is advised not to go beyond the RE frontier critical point risk level.[5]

The NFA RE equity optimizer has three ways of dealing with assuming too much risk. 1) The user is alerted with a warning message that the requested tracking error risk level is larger than the critical risk point. 2) The program provides an estimate of tracking error risk of the maximum return portfolio. 3) Only the tracking error risk optimization framework is available. In this way the user is aware of the limitations of too much assumed risk and what alternatives are available. The reason for limiting the RE equity optimization framework to tracking error risk optimization is discussed further in the following section.

The Problem with Utility Function Optimization

There is a long history of MV optimization based on specifying a "risk aversion" parameter of a quadratic utility function. Conceptually, computing the MV utility optimal portfolio, and varying the value of the utility parameter, is an alternative way of tracing the MV efficient frontier. Many MV optimizers use this technique for a variety of mostly historical reasons. Unfortunately, there are some important limitations of utility based optimization in the context of RE optimization.

Utility optimization is highly dependent on the shape of the frontier. Consider the example where the MV utility optimal portfolio has a 10% tracking error and also that the RE frontier has a critical risk point at 6% tracking error. In this case, the utility based RE optimal portfolio must have tracking error less than 6% independent of the investor's utility. Furthermore, the optimal portfolios weights will be very different.

Tracking error is the more stable and convenient portfolio selection criterion. Note that utility function optimization has the additional problem of realistically evaluating an investor's "risk aversion". Tracking error optimization transforms the investor's utility decision into a measure that has intuitive meaning for many investors. RE equity optimization uses tracking error portfolio selection so that it is always clear when an optimized portfolio is risk-return efficient and when the level of tracking error is appropriate.

[5] NFA research has confirmed in a number of cases that the in-sample critical risk point is a useful estimate of the out-of-sample critical risk point.

Other Practices

There are many additional practices commonly associated with MV optimization that are not needed, or should not be used, with RE optimization. Perhaps the simplest example is the use of many ad hoc constraints to control the MV equity optimization. But every non-financially meaningful constraint limits the ability of the RE optimizer to add investment value. By avoiding these and many other counter-productive practices, RE optimization holds the promise of substantially improving optimized portfolio performance in investment practice.

Conclusion

The illusion of portfolio risk-return efficiency associated with numerical MV optimization is a widespread fallacy of modern finance that permeates many practices of sophisticated institutions. In effect, except for the constraints in a MV optimization, nothing else really matters. Many ineffective rules and procedures in common practice come from uncorrected errors in financial journals and textbooks. But the ineffectiveness of errors is only observable in a portfolio optimization process that is otherwise investment effective. Our solution is to redefine the optimization framework so that it is not error prone. This is the course we set ourselves. We shall have more to say in future NFA newsletters.