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Defense of Markowitz-Usmen

by

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Abstract

Markowitz and Usmen (2003) (MU) compare the out-of-sample performance of Markowitz (1959) mean-variance (MV) optimization and Bayesian input estimation to Resampled Efficiency™ (RE) optimization (Michaud 1998) without enhanced estimates. In the MU study, the Michaud player always wins. Harvey et al (2008) (HLL) revisit the MU study and, using different methods for computing RE optimized portfolios, find that the Bayes and Michaud player performances are roughly tied. HLL also propose a different framework where the Bayes player always wins. We show that the first study suffers from suboptimal procedures for computing RE optimized portfolios that limits the reliability of their results. HLL's second "one-step-ahead" (OSA) test framework does not address practical out-of-sample optimization with estimation error. Revision of the MU results appears unwarranted.

Markowitz and Usmen (2003) (MU) compare the out-of-sample performance of Markowitz (1959) mean-variance (MV) optimization and Bayesian estimation to Resampled Efficiency™ (RE) optimization (Michaud 1998) without enhanced estimates. In the MU study, the Michaud player always wins in spite of inferior risk-return estimates. Harvey et al (2008) (HLL) repeat the MU study with different methods for computing RE optimized portfolios and find roughly that the Bayesian and Michaud players are tied. In addition, they propose a second study where the Bayes player always wins.

We have two basic issues with these results. First, in the HLL replay of MU, the RE optimized portfolios are not computed in the recommended way. This may explain much of the difference between the HLL and MU results. Also, HLL's "one-step-ahead" (OSA) framework is not a true out-of-sample test. In fact, it converges exactly to an in-sample test. While interesting on their own merits, HLL's results do not contradict either MU or previous work by Michaud.

Background

Markowitz (1959) MV optimization has been the standard for linear constrained efficient portfolio construction for almost fifty years. While theoretically important, MV optimization's sensitivity to estimation error in risk-return estimates is known to result in instability and likely poor out-of-sample performance. RE optimization uses resampling methods to define a more stable and investment intuitive optimality. Out-of-sample simulation tests in Michaud (1998, Ch. 6) demonstrate that RE optimization is provably effective at improving investment value on average relative to MV optimization.[1]

Markowitz and Usmen do not dispute the benefits of properly defined RE optimized portfolios relative to MV optimization. Instead, they address the relative power of Bayesian enhanced parameter estimates versus RE optimization. They develop a diffuse Bayesian procedure to improve risk-return estimates and repeat the simulation study in Michaud (1998) using Bayesian inputs for the MV optimizer investor only. Remarkably, they find that the Michaud player won on average and in each of the 30 tests performed.[2]

HLL versus MU

HLL replay the MU study by replacing the rank-order algorithm used in MU for computing Resampled Efficient Frontier™ (REF) portfolios with the λ-association method.[3] While λ-association is more compute-efficient, it is not as statistically efficient for computing RE optimized portfolios.[4] Also, HLL use 500 simulations to compute the RE optimal, λ-

[1] RE optimization, invented by Richard Michaud and Robert Michaud, is protected by U.S. and Israeli patents and patents pending worldwide. New Frontier Advisors, LLC is worldwide licensee.

[2] Michaud and Michaud (2008a, pp. 72-3) perform a simulation study with James-Stein return estimation with similar results.

[3] The λ-association procedure is discussed and compared to the rank-order algorithm recommended in Michaud (1998, Ch. 6, Appendix).

[4] λ-association has more variability and requires many more simulations for effectiveness relative to the rank-order algorithm. There are a number of additional ways to compute the RE optimal portfolios that represent a tradeoff of compute-efficiency relative to approximative power and statistical stability.

associated portfolios and 25,000 simulations to compute their Bayesian estimates. The limitations of their procedures may account for much, if not all, of their less positive results for the Michaud player in the MU game.[5]

A Second Investment Game

The HLL "one-step-ahead" (OSA) simulation framework finds that the Bayes player always wins, because it is essentially an in-sample study of relative performance. In-sample optimization, solved by Markowitz more than 50 years ago, cannot be beaten at its own game.

HLL use the predictive return distribution, instead of the true distribution, to assess average utility. In their view, the Bayes player is handicapped when future returns are dissimilar to the true though similar to the history distribution. The OSA framework ignores the true (population) parameters when estimating average utility of the optimized portfolios. Rather, each OSA test uses a predictive return distribution based solely on a sample history of a single play of the game. This predictive distribution results in expected return being very close to the sample history's mean and calculates the expected covariance matrix as the sample historical covariance matrix plus an error term.

The OSA test draws many returns from the predictive distribution of a sample history when evaluating utility of the two players. When the OSA test is repeated many times, it converges to its own distribution. As an example, if we were to evaluate out-of-sample of the maximum return portfolio, the Bayesian player will always win by their criterion. This is because the OSA will draw from a distribution where the highest expected return asset is the asset with the highest return over the sample history. Repeating the OSA test many times will, sure enough, confirm the Bayesian player's belief that the expected highest return asset is truly the highest return asset. But this is self-referential.

The question isn't whether the predictive distribution is a better estimate of future return. It is. However, there's no need to basically assume the sample history is the truth. In this case, a far better estimate of the future return distribution is available: the true distribution. A true out-of-sample test, as in MU, is the appropriate way to evaluate the usefulness of a strategy for an investor.

In addition, the appropriate measure of relative performance does not vary by kind of investor; the next period's return is as important to a short-term as a long-term investor. Only the MU framework addresses the impact of estimation error on the investment value of optimized portfolios.[6]

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[5] Given the limitations of the λ-association procedure used in their study, many more simulations were advisable.

[6] HLL use a similar argument in (Harvey et al 2006). In this earlier study, additional return distribution parameters are assumed, showing that, in-sample, the Michaud player performs less well than enhanced MV. However, as Michaud and Michaud (2008a) note, few investors are likely to be very pleased with their more "optimal" portfolios if they have less risk-adjusted return on average in the investment period. The issue of interest is whether more in-sample optimality is actually productive out-of-sample.

Conclusion

HLL does not dispute the relative benefit of properly implemented RE versus MV optimization. HLL's objective is to reassess the relative value of Bayes enhanced input estimation versus RE optimization. While the HLL replay of the MU game found essentially a tie between the Bayes and Michaud player, they use different estimation procedures that are likely to account for their results. HLL also propose a second simulation framework that finds that the Bayes player always wins. However, this second test is essentially in-sample and does not contradict the MU study conclusions.

While it is interesting to compare the relative value of Bayesian input estimation versus RE optimization, the procedures are complementary, not mutually exclusive. RE optimization properly implemented is simply an additional method available to investors for improving the investment value of optimized portfolios independent of the risk-return estimation process.

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